CRYSTAL-PHASE MODEL FOR NEUTRON-RICH ISOTOPES

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ABSTRACT. The accepted structure for a cluster of nucleons (protons and neutrons) views the cluster as a liquid, in which the wave function for each nucleon extends throughout the volume of the cluster. The mass excesses of nuclear isotopes then can be approximated by a liquid-drop model. I propose that there also exists a solid crystalline phase of nucleons, for which a cluster can be viewed as a crystal in which each nucleon is confined to one of many small potential wells in a close-packed solid structure. I assume that each small confining volume is too small to bind a second nucleon. The mass excesses of various crystal-phase isotopes then can be approximated using a crystal-phase model. The crystal-phase model is simpler than the liquid-drop model because the wave function of every crystal-phase nucleon is constrained to be the sole occupant of the small volume it occupies in the crystal lattice.

1. INTRODUCTION

The accepted structure for a cluster of nucleons (protons and neutrons) views the cluster as a liquid, in which the wave function for each nucleon extends throughout the volume of the cluster. The mass excesses of nuclear isotopes then can be approximated by a liquid-drop model. A recent calculation of zero-temperature nucleon binding [1] suggests that nuclei composed of equal numbers of protons and neutrons can be understood as fluid clusters of alpha particles. The calculation further suggests that the alpha particle fluid phase lies near a phase boundary. It has also been suggested [2] that neutron-rich nuclei may be members of this second phase, and that investigation of neutron-rich nuclei would be of interest.

I propose that there exists a solid crystalline phase of nucleons, for which a cluster can be viewed as a crystal in which each nucleon is confined to one of many equal potential wells in a close-packed solid structure. I assume that each well is just large enough to contain a single nucleon. The mass excesses of various crystal-phase clusters then can be approximated using a crystal-phase model. The crystal-phase model is simpler than the liquid-drop model because the wave function of every crystal-phase nucleon is constrained to the small volume it occupies in the crystal lattice.

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Table 1. Parameters for the liquid-drop and crystal-phase models of nuclear mass excesses (in MeV).

	Model	
Parameter	Liquid-Drop	Crystal-Phase
a_v	15.5	2.60
a_s	16.8	4.12
a_c	0.72	0.72
a_{sym}	23	0
a_p	34	0

I begin by reviewing the standard liquid-drop model and then indicate how it can be modified by removing the symmetry and pairing energy terms to obtain a crystal-phase model. The crystal-phase model predicts that sufficiently large neutron-rich clusters of nucleons are bound, and suggests avenues for future research.

2. Liquid-drop model

For ordinary nuclei, the mass excesses of the various isotopes have been approximated by a liquid-drop model in which N is the number of neutrons, Z is the number of protons, and $\mathrm{A} = \mathrm{N} + \mathrm{Z}$ is the total number of nucleons in the nucleus ${}^{\mathrm{A}}_{\mathrm{Z}}\mathrm{X}$:[1]

$$\Delta({}^{\mathrm{A}}_{\mathrm{Z}}\mathrm{X}) = \mathrm{N}\,\Delta({}^{1}_{0}\mathrm{n}) + \mathrm{Z}\,\Delta({}^{1}_{1}\mathrm{H}) - a_v\,\mathrm{A} + a_s\,\mathrm{A}^{2/3} + a_c\,\mathrm{Z}(\mathrm{Z}-1)\,\mathrm{A}^{-1/3} + a_{sym}\,(\mathrm{N}-\mathrm{Z})^2\,\mathrm{A}^{-1} - a_p\,\delta \mathrm{A}^{-3/4}, \quad (1)$$

where

$$\delta = \begin{cases} 1 & \text{Z and N both even} \\ -1 & \text{Z and N both odd} \\ 0 & \text{otherwise.} \end{cases}$$

The symbols $\Delta({}^{\mathrm{A}}_{\mathrm{Z}}\mathrm{X})$, $\Delta({}^{1}_{0}\mathrm{n})$, and $\Delta({}^{1}_{1}\mathrm{H})$ are the mass excesses of ${}^{\mathrm{A}}_{\mathrm{Z}}\mathrm{X}$, ${}^{1}_{0}\mathrm{n}$, and ${}^{1}_{1}\mathrm{H}$ respectively. The parameters a_v, a_s, a_c, a_{sym}, and a_p, as determined to fit the experimental values of $\Delta({}^{\mathrm{A}}_{\mathrm{Z}}\mathrm{X})$ [3], are listed in the second column of Table 1.

In the liquid-drop model the symmetry and pairing parameters a_{sym} and a_p are based on the idea that, in a liquid drop, neutrons and protons occupy energy levels appropriate for a spherical potential well the size of the drop, within which the wave function of each nucleon extends throughout the well.

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[1] Note $\Delta({}^{\mathrm{A}}_{\mathrm{Z}}\mathrm{X}) = c^2\,(M(Z,A) - A)$, where $M(Z,A)$ is given in equation (3.29) in [3].

3. Crystal-phase model

In the crystal-phase model, by contrast, the wave function of each nucleon is confined to a small box in a crystalline array of similar boxes, surrounded by other nucleons in their small boxes. Every box is bounded by the surrounding crystal aggregate, and every nucleon contributes to confining its neighbors. I assume that the symmetry and pairing terms do not apply to the crystal-phase model, in consequence of which the crystal-phase model and the liquid-drop model describe distinct crystal and liquid phases of nuclear material. I suggest that the crystal phase is the appropriate one for neutron-rich isotopes.

Here is the crystal-phase model:

$$\Delta({}^{\mathrm{A}}_{\mathrm{Z}}\mathrm{X}^*) = \mathrm{N}\,\Delta({}^{1}_{0}\mathrm{n}) + \mathrm{Z}\,\Delta({}^{1}_{1}\mathrm{H}) - \mathrm{a_v}\,\mathrm{A} + \mathrm{a_s}\,\mathrm{A}^{2/3} + \mathrm{a_c}\,\mathrm{Z}(\mathrm{Z}{-}1)\,\mathrm{A}^{-1/3}, \qquad (2)$$

where ${}^{\mathrm{A}}_{\mathrm{Z}}\mathrm{X}^*$ denotes a crystal-phase isotope with nucleon number A and charge Z.

Calibration. [All but the final three sentences of this subsection were lined out. John had mentioned the possibility that these experiments were not relevant for calibration.]

For the crystal-phase model (2) I have re-determined the parameters $\mathrm{a_v}$ and $\mathrm{a_s}$ in Table 1 to agree with experimental data. Recent evidence [4] has shown that the di-neutron, which is bound when confined to the nucleus ${}^{16}\mathrm{Be}$, becomes unbound when removed from ${}^{16}\mathrm{Be}$. The two neutrons then fly apart with kinetic energy 1.35 MeV. This suggests that, when a di-neutron is freed of interaction with other components of a nucleus in which it is embedded, its mass excess may be

$$\Delta({}^{2}\mathrm{n}^*) = 2\,\Delta(\mathrm{n}) + 1.35\ \mathrm{MeV}. \qquad (3)$$

Other evidence [5, 6] suggests that the tetra-neutron is either very weakly bound, or very weakly unbound. Zero binding appears to be the best currently available estimate, giving

$$\Delta({}^{4}\mathrm{n}^*) = 4\,\Delta(\mathrm{n}) + 0\ \mathrm{MeV}. \qquad (4)$$

As indicated by the asterisk superscripts in (3) and (4), I assume that ${}^{2}\mathrm{n}^*$ and ${}^{4}\mathrm{n}^*$ are both members of the crystal phase.

The experimentally determined relationships (3) and (4) are sufficient for determining the crystal-phase parameters $\mathrm{a_v}$ and $\mathrm{a_s}$ in the third column of Table 1. These values are about 1/5 of those for ordinary nuclei in the liquid-drop model, suggesting that crystal-phase isotopes may be more weakly bound than liquid-phase isotopes. It remains to determine the value of the coulomb parameter $\mathrm{a_c}$. Here I see no reason to change it from the liquid-drop value. Preliminary quantification of the crystal-phase model (2) is now complete.

TABLE 2. Mass excesses $\Delta({}^{A}_{Z}X^{*})$ for selected crystal-phase isotopes (in MeV), calculated from formula (2). An entry in **bold face** is the minimum in its row.

Nucleon Number	Nucleon Charge Z			
A	0	1	2	3
14	100.53	99.75	**99.56**	99.98
13	93.91	93.12	**92.95**	93.40
12	87.25	86.47	**86.31**	86.79
11	80.56	79.78	**79.65**	80.16
10	73.84	73.05	**72.94**	73.49
9	67.07	66.29	**66.20**	66.80
8	60.25	59.47	**59.41**	60.06
7	53.38	52.59	**52.56**	53.29
6	46.43	**45.65**	45.66	46.46
5	39.40	**38.62**	38.68	39.58
4	32.27	**31.48**	31.61	32.64

Decay modes. Table 2 lists the mass excesses $\Delta({}^{A}Z^{*})$ for selected neutron isotopes and selected neutron-rich nuclei, based on the crystal-phase formula (2) rounded to two digits following the decimal point. Examination of this table reveals the potential for a number of exothermic reactions, including beta decays (all isotopes to the left of a bold-faced entry) and electron captures (all isotopes to the right of a bold-faced entry).

[A number of changes were suggested to the exposition leading up to (5).] Table 2 also shows that neutron isotopes ${}^{A}n^{*}$ containing $A \geq 5$ neutrons are bound and stable against decay to A free neutrons which together have mass excess 8.071A MeV. However they are not stable against beta decay in which a neutron changes into a proton. An example is

$$ {}^{10}n^{*} \longrightarrow {}^{10}H^{*} + 0.79 \text{ MeV} \tag{5}$$

in which the proton in ${}^{10}H^{*}$ is bound equally strongly as the neutron it replaces, and the energy released is simply the mass difference between n and ${}^{1}H$. The ${}^{10}H^{*}$ released in (5) is capable of a second beta decay to a doubly charged member of the crystal phase:

$$ {}^{10}H^{*} \longrightarrow {}^{10}He^{*} + 0.11 \text{ MeV}. \tag{6}$$

The energy released in (6) is smaller than that released in (5) because of the coulomb energy accompanying addition of the second proton.

Beta decay of $^{10}\mathrm{He}^*$ is endothermic because the coulomb energy increase would be too great. However $^{10}\mathrm{He}^*$ can decay by emission of an ordinary alpha particle ($^4\mathrm{He}$),

$$^{10}\mathrm{He}^* \longrightarrow {}^6\mathrm{n}^* + {}^4\mathrm{He} + 24.09 \text{ MeV}, \tag{7}$$

together with the neutral crystal isotope $^6\mathrm{n}^*$. The $^{10}\mathrm{He}^*$ can also decay by emission of an ordinary tritium atom ($^3\mathrm{H}$),

$$^{10}\mathrm{He}^* \longrightarrow {}^7\mathrm{H}^* + {}^3\mathrm{H} + 5.40 \text{ MeV}, \tag{8}$$

together with the crystal isotope $^7\mathrm{H}^*$.

4. Summary

The calculated mass excess values in Table 2 are approximations based on uncertain values of the $^2\mathrm{n}^*$ and $^4\mathrm{n}^*$ di-neutrons and tetra-neutrons. They are not expected to be reliable. Yet they are valuable because they support the idea that a nuclear crystal phase may actually exist, and that a crystal-phase model may be made more reliable through further study.

The experimental binding energies for di-neutrons and tetra-neutrons point to the existence of a solid crystalline phase of nuclear matter distinct from the familiar liquid phase. A crystal-phase model is proposed for the masses of crystal-phase nuclei.

References

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